# Individual Differences in Subtractive Thinking and Creative Problem-Solving: Evidence for an Association with Convergent but not Divergent Thinking

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**Data, Code, and Other Materials**

Available at https://osf.io/sn2yu/

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**Disclosure Statement**

The authors report that there are no competing interests to declare.

**Abstract**

Recent research suggests that people overlook the option to remove elements when making decisions or solving problems, preferring instead to make additive changes; however, little research has explored individual differences in this tendency to make subtractive changes. This study investigated the relationship between individual differences in creative thinking (via convergent and divergent thinking), and the tendency to suggest subtractive changes. Two-hundred fifty-two participants completed a sample of subtractive, convergent, and divergent thinking tasks. Only convergent thinking was weakly but significantly predictive of subtractive thinking. These results suggest that processes involved in creative cognition are related to subtractive thinking. More research is needed to refine the measurement of individual differences in subtractive thinking as well as to clarify the specific processes underlying the relationship between creative and subtractive thinking.

*Keywords:* additive bias, subtractive thinking, creativity, problem solving

## Introduction

The way in which we go about solving problems depends, in part, on our *problem representation* (Newell & Simon, 1972)—our conception of the problem space, its elements, and the types of transformations that can occur. If our problem representation is flawed or incomplete, then success is unlikely. Importantly, a common source of errors in correctly representing a problem is the inappropriate application of prior knowledge; indeed, we often need to overcome default representations and approaches when tackling novel challenges (Ohlsson, 2011; Smith, 1995; Ward, 1994).

Recent research has begun to explore the default problem representations individuals construct when they are tasked with making or suggesting changes, e.g., to solve a problem or improve a situation. Specifically, people tend to add new elements as opposed to subtract old elements—even when a subtractive option is optimal (Adams et al., 2021). However, less is known about individual differences in the tendency to use subtractive vs. additive transformations (cf., Juvrud et al., 2024). The goal of the present research was to investigate individual differences in subtractive thinking, specifically, whether creative thinking measures account for variability in subtractive thinking.

We use *subtractive thinking* to refer to mental processes that lead one to remove elements in order to generate an improvement or solve a problem when adding or rearranging elements are viable options. A recent series of studies by Adams et al. (2021) explored the tendency to overlook subtractive changes. Adams and colleagues presented participants with a wide variety of situations in which elements could have been subtracted (e.g., to change a recipe). Across all prompts, subtractive changes were far more numerous than additive changes, though, Adams et al. found that providing cues highlighting subtractive changes as a *possibility* significantly increased the number of subtractive changes made or suggested by participants. Adams et al. (2021) also found that subtractive changes were more likely when cognitive load was low rather than high. A possible explanation for these experimental results is that addition is the default approach individuals take in situations where non-subtractive changes are viable options—overcoming this default is cognitively demanding, thus the bias against subtractive solutions.

Although*,* normatively, individuals may fail to use subtraction, there is variability. The primary question we sought to investigate is whether variability in subtractive thinking is associated with creative thinking. Creativity refers to the ability to generate novel solutions and ideas that are goal relevant (Runco & Jaeger, 2012). There is a considerable body of work suggesting novel ideas do not come to mind readily, but that we are constrained by our prior knowledge, memories, and habits (Ohlsson, 2011; Storm et al., 2020)—creative individuals, therefore, are those who are better able to overcome interference from routine or default modes of thinking and behaving. As argued above, non-subtractive approaches may be the default and therefore creative thinking may be useful in overcoming this routine way of thinking.

Creative thinking is often partitioned into divergent and convergent thinking. Measures of divergent thinking (DT) assess the ability to produce multiple diverse ideas. Successfully generating many creative ideas requires consideration of multiple perspectives and approaches. Because it is apparent from Adams et al. (2021) that subtractive solutions are uncommon, it is reasonable to expect that divergent thinking can be useful in generating subtractive solutions.

Convergent thinking (CT) refers to the ability to identify and select a single correct (or best) solution. Typically, in CT tasks, the path to a solution is not obvious; one must overcome an initial incorrect assumption or otherwise change one's problem representation (Wiley &

Danek, 2024). To the extent that thinking of *removing* rather than *adding* involves a shift in a problem representation, measures of CT may be predictive of subtractive thinking.

## Current Research

In the present research, we aimed to 1) conceptually replicate the additive bias found by Adams et al. (2021) using similar tasks and 2) explore whether variation in creativity-related measures predict subtractive thinking. Participants completed several tasks intended to measure DT, CT, and subtractive thinking. We expected that either or both DT and/or CT would be predictive of subtractive thinking.

# Method

## Participants

Participants ($N$ = 265) were recruited from Prolific, an online participant recruitment platform, into a larger study investigating creative cognition. Of those, 252 participants attended all sessions and were included in the present study. We intended to recruit 250 individuals into this portion of the study, a sample size sufficient to identify a small effect.

Participants were recruited from the U.S. and self-reported being between the ages of 18-35 ($M$ = 26.95, $SD$ = 4.98), 52% female, and fluent in American English. All participants were required to use a desktop or laptop computer and to complete their sessions on Zoom, a teleconferencing platform, with a proctor.

## Materials

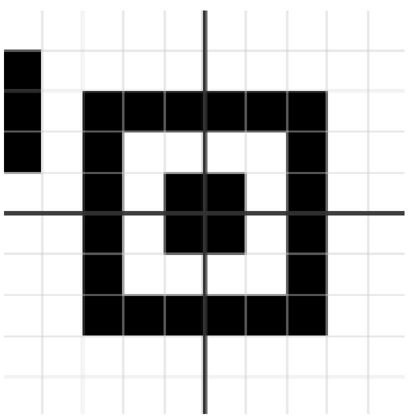
We provide information about all tasks used in this study below. To see demonstrations of these tasks, please refer to the online project repository: https://osf.io/sn2yu/.

***Subtractive Thinking Tasks*** *(Adapted from Adams et al., 2021)*

**Grids.** Each item in the Grids task required the participant to use the fewest mouse clicks to shade/unshade rectangles to make figures symmetrical (See Figure 1). For each figure, the most efficient strategy was to "subtract" rather than "add" shaded squares. A participant's score reflected the total number of responses which subtracted from the grid rather than added to it. There were five grids, thus the maximum score was 5.

**Figure 1**

*Grids Task*



**Lego Task.** This task presented participants with images of a Lego structure, shown at four different angles. The objective was to describe how to change the structure to balance a brick on top of it. Participants were asked to describe how they would change the structure as well as how many Lego blocks they would need, if any, to achieve the goal. The maximum score was 1, reflecting whether the described change(s) was/were subtractive or not.

**Mini Golf Task.** Participants were presented with an image of a miniature-golf course and were asked to suggest ideas to improve the course. Participants were given 3 minutes to list as many ideas as possible. Scores on this task reflected the number of subtractive changes suggested.

### *Divergent Thinking Tasks*

For all DT tasks, participants were instructed to think of creative, novel, or unusual ideas and that they should attempt to be as creative as possible. After each task, participants were shown their ideas and were asked to select their most creative idea(s) (Silvia et al., 2008).
All DT tasks were scored subjectively by three trained raters. Raters were asked to rate each response globally on a five-point scale—for more details, see https://osf.io/sn2yu/.

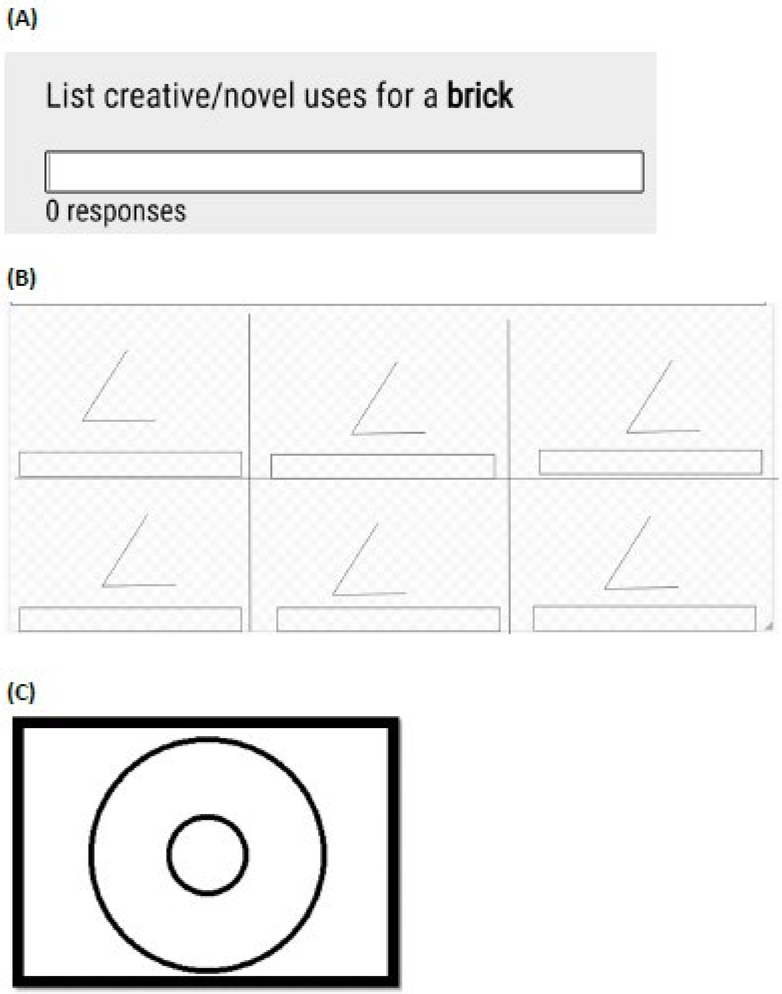
**Alternate Uses Task (AUT).** Participants were instructed to think of alternative uses for two items (brick and paperclip; see Figure 2A). Participants had 3 minutes per item. After completing the task, they selected their top two creative ideas for each object.

**Picture Completion.** This task was administered via Google Drawings. Participants were shown simple incomplete shapes and were asked to use the scribble function to complete the shapes and their keyboard to type a label or caption (see Figure 2B). They were instructed that the drawings did "not need to be beautiful" but that they should be creative. Participants were given 5 minutes to generate up to six creative drawings. Upon completion, participants selected their top two most creative ideas.

**Doodles** (Nishimoto et al., 2010; Severson et al., 2005)**.** This task consisted of developing descriptions for abstract drawings (see Figure 2C). The task consisted of three doodles and participants spent 2 minutes on each creating labels/descriptions. Upon completion, they selected their most creative label/description for each doodle.

**Figure 2**

*Divergent Thinking Tasks*



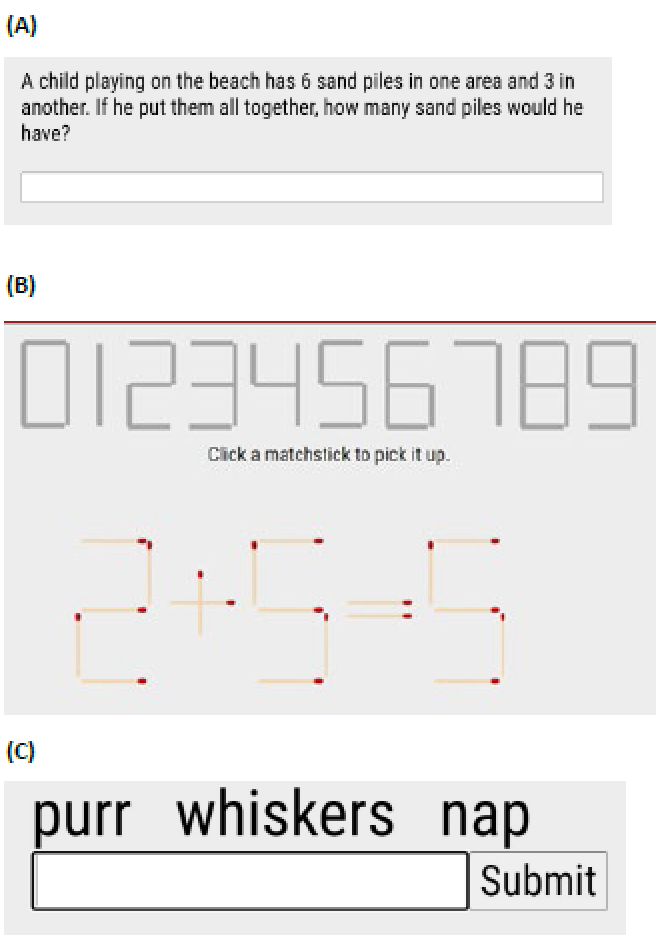

***Convergent Thinking Tasks***

**Riddles.** This task consisted of 21 critical items, largely drawn from Dow and Mayer (2004). Each item required the participant solve a riddle or puzzle (see Figure 3A). Participants had up to 10 minutes to complete the task.

**Matchstick Math.** The Matchstick Math task (inspired by Knoblich et al., 1999) presents participants with false mathematical statements made of matchsticks; participants are asked to move a single matchstick to make the statement true (see Figure 3B). The task consisted of five items with feedback and five items without feedback. In part one, participants had 2 minutes and 45 seconds to complete each item and received up to two hints as time passed. In part two, participants had 2 minutes to complete each item and received up to one hint as time passed. Participants received between 1-3 points in part one, and between 1-2 points in part two, depending on the number of hints they viewed. The maximum score was 25.

**Figure 3**

*Convergent Thinking Tasks*



**Remote Associates Test (RAT).** The RAT (Mednick, 1962) presents participants with three problem words linked to a common fourth solution word (see Figure 3C). Participants were given 8 minutes to complete 24 critical RAT items.

## Procedure

The subtractive thinking, DT, and CT tasks were part of a larger study. Participants completed these tasks along with many others in three separate sessions. During their sessions, participants logged on to a Zoom teleconference meeting, with audio, video, and screen-sharing enabled. Participants were briefly introduced to the proctor who explained expectations for the study and guided them to our proprietary data collection website and a menu of tasks. Participants received the tasks used in the present study in the following order: AUT, Grids, Riddles, RAT, Lego, Matchstick Math, Mini-Golf, Doodles, Picture Completion.

## Planned Analyses and Results

First, we investigated whether we observed an additive bias in our versions of the Grids, Mini Golf, and Lego tasks (see Adams et al., 2021). Descriptive statistics and test results are presented in Table 1—for each of these tests, the null was that there was an equal probability of producing additive and subtractive changes and all tests were two-sided.

**Table 1**

*Descriptive Statistics for the Subtraction Tasks and Results of Analyses*

| | *n* | *M* | *SD* | Skew | Kurtosis | Test | *p* |
|---|---|---|---|---|---|---|---|
| Grids | 251 | 0.75 | 0.34 | -1.21 | 0.08 | Wilcoxon signed rank test | <.001 |
| Mini-Golf | 251 | 0.35 | 0.48 | 0.60 | -1.64 | Exact binomial test | <.001 |
| Lego | 249 | 0.20 | 0.4 | 1.52 | 0.30 | Exact binomial test | <.001 |

As can be seen in Table 1, participants produced significantly *fewer* subtractive changes for both the Mini Golf and Lego tasks; in contrast, for the Grids task, participants produced significantly *more* subtractive changes.

Having observed the additive bias in two out of three tasks, we next investigated whether creative cognition is associated with subtractive thinking. First, we standardized all scores and formed subtractive thinking, DT, and CT composites with all available data. Next, we investigated whether there were any outliers in the composites—two subtractive thinking scores were replaced with the cut-off value of 3.33 standard deviations from the mean. We then conducted a regression analysis with the subtractive thinking composite as the outcome variable and DT and CT composites as predictors.

Descriptive statistics for the subtractive thinking tasks are provided in Table 1 (above); Table 2 presents descriptive statistics for the predictors. Correlations amongst all measures are provided in Table 3. Of note, individual subtractive thinking tasks shared little variance with other tasks or with each other, however, the subtractive thinking composite is weakly correlated with the CT composite ($r$ = .19, $p$ = .003), though not significantly correlated with DT ($r$ = .09, $p$ = .149).

**Table 2**

*Descriptive Statistics for the Divergent and Convergent Thinking Tasks.*

| | n | M | SD | Skew | Kurt. | IC | IC Type |
|---|---|---|---|---|---|---|---|
| AUT | 249 | 1.85 | 0.51 | 0.77 | 0.41 | .78 | ICC3 |
| Doodles | 252 | 1.91 | 0.54 | 0.75 | 0.01 | .76 | ICC3 |
| Pic | 241 | 1.61 | 0.57 | 0.99 | 0.48 | .75 | ICC3 |
| RAT | 252 | 0.64 | 0.13 | -0.35 | -0.12 | .63 | alpha |
| Riddles | 250 | 0.45 | 0.17 | -0.12 | -0.38 | .73 | alpha |
| Matches | 251 | 11.7 | 2.83 | -0.89 | 2.09 | .67 | alpha |

*Note.* Kurt. = kurtosis; IC = internal consistency.

**Table 3**

*Correlations Among all Measures.*

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 AUT | | | | | | | | | | | | | |
| 2 Doodles | **.40** | | | | | | | | | | | | |
| 3 Pic | **.18** | **.19** | | | | | | | | | | | |
| 4 RAT | .10 | **.13** | **.24** | | | | | | | | | | |
| 5 Riddles | **.19** | **.20** | **.20** | **.40** | | | | | | | | | |
| 6 Matches | **.20** | **.15** | **.15** | **.25** | **.44** | | | | | | | | |
| 7 Grids | .12 | .11 | .06 | **.16** | **.15** | **.15** | | | | | | | |
| 8 Golf | -.12 | -.02 | .04 | .06 | .04 | -.06 | -.02 | | | | | | |
| 9 Golf01 | **-.13** | -.05 | .01 | .10 | .07 | .00 | -.03 | **.81** | | | | | |
| 10 Lego | .06 | .06 | .09 | .08 | .08 | .12 | -.02 | .07 | .05 | | | | |
| 11 DT | **.75** | **.75** | **.65** | **.21** | **.28** | **.23** | **.14** | -.05 | -.08 | .09 | | | |
| 12 CT | **.22** | **.21** | **.25** | **.72** | **.80** | **.75** | **.21** | .02 | .07 | **.13** | **.32** | | |
| 13 Sub | .03 | .09 | .09 | **.18** | **.16** | .10 | **.69** | **.65** | **.53** | **.32** | .09 | **.19** | |
| 14 Sub012 | -.06 | .00 | .06 | **.13** | .11 | .07 | -.03 | **.66** | **.78** | **.66** | **.00** | **.14** | **.79** |

*Note.* Bolded values are significant at the .05 level. AUT = Alternate Uses Task; Pic = Picture Completion; RAT = Remote Associates Task; Matches = Matchstick Math; Golf = Mini Golf task; Golf01 = dichotomized Mini-Golf task used in exploratory analyses; DT = divergent thinking composite; CT = convergent thinking composite; sub = subtractive thinking composite; Sub012 = subtractive thinking composite used in exploratory analyses.

The results of the regression analysis are presented in Table 4. Notably, the CT composite was a significant predictor of subtractive thinking ($\beta$= .18, $p$ = .005), however, DT was not ($\beta$ = .02, $p$ = .587).

**Table 4**

*Regression Model Predicting Subtractive Changes as a Function of Convergent and Divergent Thinking.*

| | *b* | *β* | *SE* | *t*-value | *p*-value |
|---|---|---|---|---|---|
| Intercept | 43.9 | NA | 2.50 | 17.57 | <.001 |
| CT | 0.11 | 0.18 | 0.04 | 2.71 | .007 |
| DT | 0.02 | 0.04 | 0.04 | 0.53 | .594 |

*Note*. Model $R^2 = 3.68\%$, $F(2,249) = 4.753$, $p = .009$

## Exploratory Analyses and Results

The results of the planned analyses suggest an association between CT and subtractive thinking, however, there were two issues we wished to address in our exploratory analyses (see the Discussion section below for other issues). First, the Grids task was the only subtractive thinking task to be significantly correlated with the CT measures, raising the possibility that the subtractive thinking composite correlates with CT because of this one task.

A second issue is that, in the Mini-Golf task, participants received points for *every* subtractive idea they provided; however, providing a single subtractive idea may be evidence that the additive bias was overcome and thus assigning points for each additional subtractive idea is redundant. Thus, in our exploratory analyses, we assigned participants scores of 1 or 0 to indicate whether they provided at least one subtractive idea or not, respectively. To form our new subtractive thinking composite, we added the dichotomized Mini-Golf score and Lego score together; consequently, the composite could take on the values of 0,1, or 2.

Here we present the results of our final exploratory analysis; for intermediary analyses, see the OSF project page (https://osf.io/sn2yu/). Table 3 (above) presents correlations between the dichotomized Mini-Golf task, newly formed subtractive thinking composite, and all other variables. Table 5 presents the results of a Poisson regression analysis with the CT and DT composites as predictors and the newly formed subtractive thinking composite as the outcome variable. As can be seen in Table 5, CT was a significant predictor ($b$ = .02, $p$ = .047), however, DT was not ($b$ = -.01, $p$ = .505).

**Table 5**

*Poisson Regression Model Predicting Subtractive Changes as a Function of Convergent and Divergent Thinking.*

| | *b* | *SE* | *t*-value | *p*-value |
|---|---|---|---|---|
| Intercept | -1.42 | 0.74 | -1.91 | .057 |
| CT | 0.02 | 0.01 | 1.98 | .047 |
| DT | -0.01 | 0.01 | -0.67 | .514 |

## Discussion

Efficiently solving problems requires understanding the elements of the situation at hand, and consideration of how those elements can be manipulated. In many situations it appears that people preferentially focus on transformations that add rather than remove existing elements (Adams et al., 2021). The goal of this study was to replicate Adams et al. while testing whether individual differences in creative thinking would be predictive of the tendency to make subtractive changes.

We successfully replicated the additive bias identified by Adams et al. (2021) in two out of three tasks administered in the present study. Participants tended to suggest additive rather than subtractive solutions in both our versions of the Mini-Golf and Lego tasks; however, participants produced significantly more subtractive solutions in the Grids task (cf. Adams et al., 2021, Supplementary Information, Study 1).

It is possible that our failure to observe the additive bias in the Grids task is due to random chance or other factors, however, we wish to highlight two potential reasons for the difference. First, our participants were recruited into a paid multi-session study in which they completed sessions while on a teleconferencing platform. As a result, participants may have been more engaged and attentive. A second issue is that our participants knew they were taking part in a study on creative thinking; as a result, our participants may have been primed to think creatively or to look for “tricks”. We favor the first explanation. Anecdotally, the Grids task is a simple task—attentive individuals *should* be able to identify subtraction as the most efficient method. Additionally, we were able to replicate the additive bias in the two other subtractive thinking tasks.

Turning to our investigation of individual differences in subtractive thinking, we observed that *convergent*, but not *divergent*, thinking is associated with subtractive thinking. This suggests that thinking of subtractive solutions may be somewhat akin to changing a problem representation (Newell & Simon, 1972). In CT tasks, prior knowledge and experiences impede awareness of uncommon solutions; success in CT tasks often involves shifting from default approaches and reinterpreting problems (Wiley & Danek, 2024). The ability to break free of default approaches may underlie performance on both CT and subtractive thinking tasks. Such an interpretation is consistent with work suggesting that people with greater attention control show better performance on insight problems (Cushen & Wiley, 2011).

One might expect that individuals with greater DT performance would be more likely to produce subtractive solutions because they tend to explore a wider range of ideas. However, it may be that DT tasks are relatively poor measures of the cognitive processes required to radically transform problem representations. This may be because of measurement error (e.g., unreliability of ratings) but it may also be a feature of DT. In DT tasks, individuals can generate ideas through automatic associative processes (Beaty & Kenett, 2023); for example, thinking of a brick leads to thinking of its red hue which reminds one of seeing a red stain left by a shard of brick, ultimately leading one to suggest using a brick “like a piece of chalk to make art.” Associative processes, however, may be less likely to lead individuals to shift their mode of thinking in subtractive thinking tasks which, by our definition, elicit the *viable* default approach of adding or rearranging elements. While individuals can use controlled processes to transform their representations in DT tasks (Beaty & Kenett, 2023), CT tasks appear to be especially dependent on representational change.

Additional limitations to the present research should be noted. First, our measures of subtractive thinking did not share much variance with each other or with other tasks. The observed correlations may be due to the limited number of items in any one subtractive thinking task or the difficulty of the tasks (which will reduce variability), but it may be because the variability in our subtractive thinking tasks largely reflects *noise*. Our use of composites was intended to curtail some of the psychometric issues inherent in these tasks, however, correlations amongst the individual tasks were still smaller than expected. Researchers interested in individual differences in subtractive thinking should use multiple existing measures to form composites or develop novel measures with better psychometric properties.

A second limitation of the present study is that we investigated a limited number of variables and therefore cannot specify why exactly convergent thinking is related to subtractive thinking. Our theory is that processes involved in creative cognition should support subtractive thinking, however, creative cognition is subserved by a number of basic cognitive processes. The fact that convergent, but not divergent, thinking was associated with subtractive thinking suggests some possibilities for future investigation (e.g., attention control; Cushen & Wiley, 2011).

In summary, we found that the tendency to neglect subtractive changes is a fairly robust phenomenon. Moreover, subtractive thinking appears to be sensitive to individual differences in higher-level cognitive processing. Specifically, individual differences in convergent, but not divergent, thinking was predictive of the tendency to subtract elements in situations where the addition or transformation of existing elements was a viable option.

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